MAT122

Meaning of the Derivative

Leibniz Notation

$$f'(x) = \text{instantaneous rate of change of } f \text{ at } x.$$

so far $f'(x)$ has been used to represent the derivative

$$f'(x) \approx \frac{\Delta y}{\Delta x} = \frac{dy}{dx}$$

Units of a Derivative

Velocity is an example of a derivative.

ex. The cost C in dollars of building a house A square feet in area is given by the function $C(A)$.

What are the units of $C'(A)$?

ex. If $q = f(p)$ gives the number of pounds of sugar produced when the price per pound is p dollars. What are the units of $\dfrac{dq}{dp}$?

What is the interpretation of $\left.\dfrac{dq}{dp}\right|_{p=3} = 50$?

50

Interpretation

ex. The time, L, in hours that a drug stays in the system is a function of the quantity, q, administered in mg.

    a. Interpret $L(10) = 6$

    b. Write the derivative in Leibniz notation.

    c. If $L'(10) = 0.5$, what are the units of 0.5?

    d. Interpret $L'(10) = 0.5$ in terms of dose and duration.

Second Derivative

Since a derivative is a function, we can calculate **its** derivative.

For the function f:
       the derivative of its derivative, $f\,'$, is called the **second derivative** and is denoted as $f\,''$.

In Leibniz notation: the derivative of the derivative, $\dfrac{dy}{dx}$, is $\dfrac{d^2y}{dx^2}$.

Meanings of Derivatives

Increasing/Decreasing

Concavity

$f' > 0$

$f' < 0$

$f'' < 0$
Concave down

Concave up
$f'' > 0$

$f' < 0$

$f' > 0$